

## Multi-Terminal Impedance Measurements, or... Why Do Those Bridges Use So Many Connections?

This application note explains how the use of multiple connections can reduce or remove impedance measurement errors caused by series impedance in the connections or shunt impedance across the unknown. These techniques are important not only for precision laboratory measurements, but also in production testing, circuit troubleshooting, and process control. A particularly critical application is that of "in-situ" or in-circuit testing where the component to be measured is soldered on a printed wiring board and thus connected to many other components that could affect its measurement severely. Many bridge users are familiar with 3- and 4-terminal techniques. Several new instruments use five connections, and some test systems use six. This note will start with two and work up to eight. First we should make it clear that in the arithmetic of counting terminals:

$$1 \text{ terminal} = 1 \text{ wire} = 1 \text{ lead} = 1 \text{ connection}$$

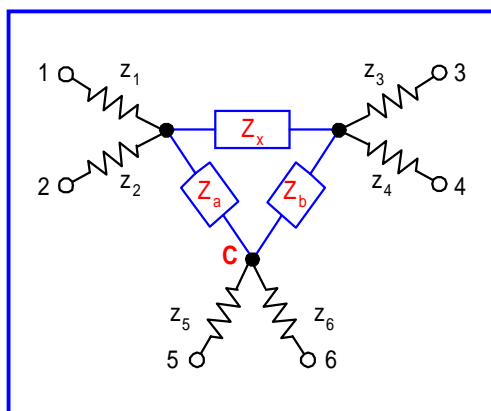
We only count the terminals at one end of a connecting wire: a 2-terminal bridge connected to a 2-terminal resistor by two wires makes a 2-terminal or 2-connection measurement. The terminals may be binding posts, wire leads, or pins in a connector; the arithmetic is the same.

To illustrate measurements with various numbers of terminals, we shall describe "ideal" measurements that make use of ideal voltmeters and ammeters. These meters measure DC or complex AC (i.e., real and imaginary parts), have absolute accuracy, and dissipate no power (the voltmeter has infinite impedance and the ammeter has zero impedance). Such meters, unfortunately, are not commercially available, and although modern digital meters can do quite well, most precise measurements use quite different techniques. The ideal-meter methods define each type of measurement and its theoretical result. Practical methods and actual instruments may use entirely different components and circuit configurations, but they make the same type of measurements if they get the same result (or at least very close to it).

For example, a 4-terminal bridge is a good 4-terminal bridge if it gets the same result as the ideal-meter 4-terminal method would have gotten if it could have been used. We also need a standard "unknown", a device to be tested that illustrates the capabilities of the various ideal measurement methods. The circuit chosen (Figure 1) has six terminals, that is, six places where connections may be made. It has eight impedances besides the impedance  $Z_x$  whose value we desire. The choice of this circuit is not a random one, for this particular network represents several practical situations. For example, all connections will have series impedance, and therefore we might as well show some impedance (lower case z's shown as resistors) in series with each terminal.

## Impedance Delta Network

The inner delta network of  $Z_x$ ,  $Z_a$ , and  $Z_b$  can represent several situations. It might be a three-terminal standard capacitor where  $Z_a$  and  $Z_b$  represent stray capacitance to the case or guard (point C) or it might represent the impedance,  $Z_x$ , buried in a network where  $Z_a$  and  $Z_b$  represent actual circuit components shunting it.



**Figure 1: Example Delta Network**

This inner delta network could also represent any passive three-terminal network whose effective delta circuit value (short circuit transfer impedance) is desired. The reason for using more and more connections is to try to measure  $Z_x$  with no errors resulting from the other eight impedances. In most cases there will be errors. In general, the measured impedance,  $Z_m$ , could be expressed in the form:

$$Z_m = \frac{Z_x (1 + A) + B}{1 + CZ_x}$$

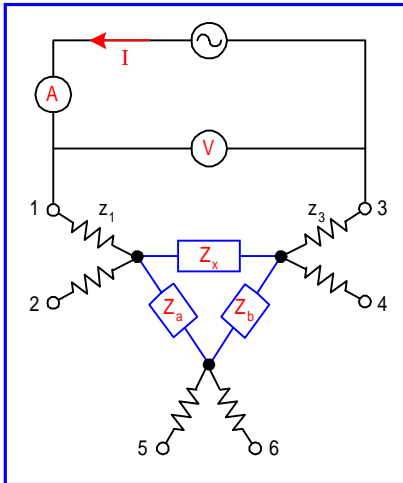
where A, B and C are functions of these other eight impedances. If the measurement is accurate to any reasonable degree, A,  $B/Z_x$ , and  $CZ_x$  must be reasonably small compared to unity. If so, we can write:

$$Z_m \cong Z_x \left( 1 + A + \frac{B}{Z_x} - CZ_x \right)$$

and the approximation is a very good one. We'll use this form for simplicity and neglect second order errors. Note that to get the errors in percent, the error terms should be multiplied by 100%. We'll see that if we use enough terminals, and some more imaginary devices, we can make a measurement on this test circuit with no error at all.

## Two-Terminal Measurements

Ohm's Law defines the impedance of a device as: the ratio of the voltage across the device to the current through it.  $Z=V/I$ . This requires at least two connections and therefore the arithmetic of terminals starts with two. With only two terminals, the same terminals must be used for both applying a current and measuring a voltage as shown in Figure 2.



**Figure 2: 2-Terminal Measurement**

$$Z_m = \frac{V}{I} = Z_x \left( 1 + \frac{z_1 + z_3}{Z_x} - \frac{Z_x}{Z_a + Z_b} \right)$$

$$Z_x = \text{series lead error} \quad Z_a + Z_b = \text{shunt loading error}$$

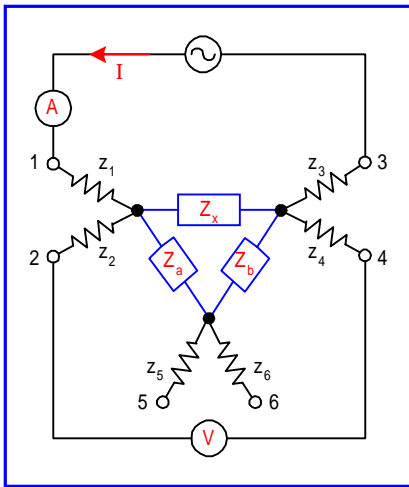
When our test circuit is measured in this way, the result may not be an accurate measurement of  $Z_x$  as shown. ( $V$  and  $I$  are the ideal voltmeter and ammeter readings.) There are two types of errors, and these are the errors that measurements with more connections attempt to avoid. The series impedances  $z_1$  and  $z_3$  add to the measured value of impedance. They cause an error that is important when  $Z_x$  is small. They add a fixed error in ohms (as opposed to percent) for their effect could be expressed as  $Z_m = Z_x + z_1 + z_3$ . If  $Z_x$  could be shorted we could measure  $z_1$  and  $z_3$  and correct for them. Analogously, the shunt impedance of  $Z_a$  and  $Z_b$  in series decreases the measured impedance. Their effect is most important when  $Z_x$  is large or its reciprocal  $Y_x$  (admittance) is small. We could express their error as:

$$Y_m = \frac{1}{Z_m} = Y_x + \frac{1}{Z_a + Z_b}$$

If we could open-circuit  $Z_x$ , we could measure the effect of  $Z_a$  and  $Z_b$  and correct for it. Two terminal measurements are adequate for a great many measurements. While many new instruments are capable of more sophisticated measurements, in many cases this capability is neither required nor used. Better techniques are required when the impedance to be measured is very high or very low, when high precision is required, or when series impedances or shunt admittances are unusually large.

## Four-Terminal Measurements

Strangely enough, 4-terminal measurements are simpler to explain than 3-terminal ones and therefore we'll jump from two to four. With a second pair of terminals available, we can measure voltage between one pair and apply current to the other as shown in Figure 3. This simple improvement removes the series lead impedance error completely and makes the four-terminal method ideal for measurements on low impedances that are negligibly affected by the remaining shunt impedance error. Unfortunately, for AC, the diagram (Figure 3) is not complete.



**Figure 3: Four-Terminal Measurement**

$$Z = \frac{V}{I} = Z_x \left( 1 - \frac{Z_x}{Z_a + Z_b} \right)$$

There will be some mutual inductance ( $M$ ) between the current leads and the voltmeter leads that will add a voltage  $\pm j\omega M$  to the voltmeter reading and  $\pm j\omega M$  to the measured impedance. Note that the mutual inductance may have either sign depending on the lead geometry. This mutual inductance may be kept small by twisting the input current leads together, twisting the voltmeter leads together, or by doing both. It is even better to use two coaxial cables, one for the two current connections and one for the two voltage connections. Four-terminal instruments and standards often have the terminals labeled as either "current" terminals or "potential" (voltage) terminals. In theory these terminal pairs are completely interchangeable.

However, there are two situations where they are not. The first is when the current applied is very large as it might be in a four-terminal "shunt" used to measure a high current. The potential terminals aren't designed for such duty. The second situation involves the measurement device that may be more immune to lead impedance in some of its connections than in others. Usually its potential terminals can tolerate more impedance than its current terminals, although this is not always the case. The operating instructions for each specific instrument should be considered.

## Four-Terminal Kelvin Measurement

Four-terminal measurements have been implemented in many ways and a few are illustrated in Figure 4. The most famous of these is the Kelvin Bridge (Figure 4a) that is widely used for precision, DC resistance measurements. It has two additional bridge arms ( $R_a$  and  $R_b$ ) to balance the voltage drop in the "Yoke" connection. This circuit has associated Lord Kelvin's name so closely with the 4-terminal technique that we often use the term "Kelvin" in place of "four-terminal". We may talk of a Kelvin measurement or a Kelvin connection even though we don't mean to refer to the actual Kelvin bridge circuit.

The potentiometer method (Figure 4b) uses several voltage ratio measurements and a calculation. A 4-terminal impedance can also be determined from a series of four 2-terminal measurements. Generally, successive measurements with simple connections and appropriate calculations can give the same results as the more complicated methods. For example,  $Z_x$  in Figure 1 may be determined exactly by making nine 2-terminal measurements or three 4-terminal ones.

With AC, transformers may be used to inject compensating voltages in appropriate places to reduce errors as is done in the circuit of Figure 4c. The voltage drop ( $E_y$ ) in the connection between the low impedance ratio arm  $R_a$  and the unknown  $C_x$  is injected between the other two high-impedance arms thus canceling its effect. Most new, four-terminal instruments use precision, high-input-impedance differential amplifiers to allow the measurement of the voltage across the unknown with negligible effect from series impedance (Figure 4d).

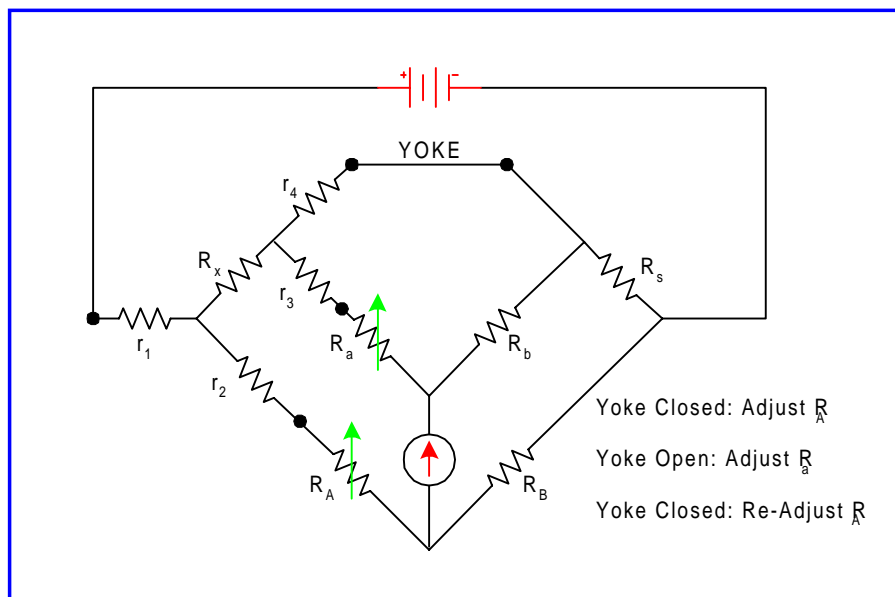
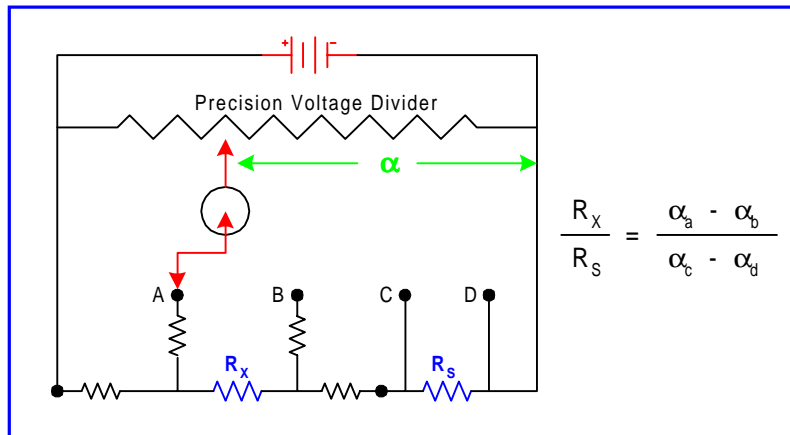
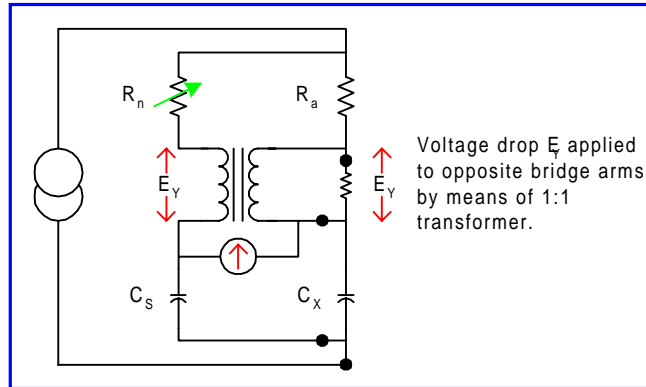


Figure 4a: Kelvin Bridge

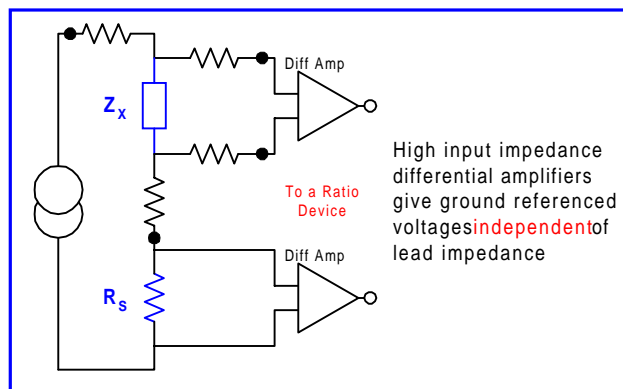
## Four-Terminal Measurements



**Figure 4b: Potentiometer Method**



**Figure 4c: Transformer Compensation**

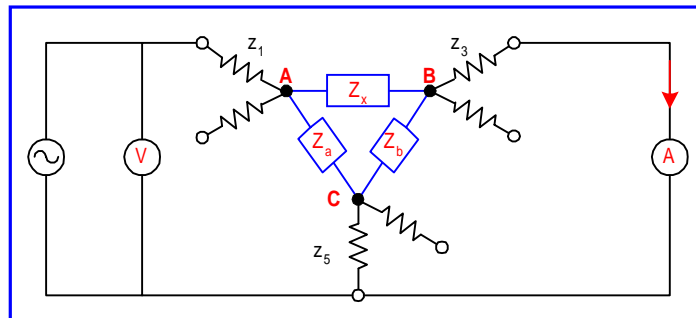


**Figure 4d: Differential Amplifiers**

## Three-Terminal Measurements

While the four-terminal measurement applies a current and measures the resulting open-circuit voltage, the three-terminal measurement does just the opposite; it applies a voltage and measures the resulting short-circuit current (Figure 5). To do this, it uses an extra terminal: a common connection between the input voltage and output current. Impedances from either terminal of  $Z_x$  to this third terminal have no effect (if the series lead impedances are zero). Impedances will load the input, but even if it reduces the voltage, the voltmeter reading is still valid. Impedance shunting the ideal ammeter has no effect because the ammeter has zero impedance.

This third terminal is called the **guard**. The effect of any stray path, capacitive or conductive, (shunting  $Z_x$ ) can be removed by intercepting it with a shield tied to the guard point. Likewise, "shunting  $Z_x$ " can effectively be removed in a series string of actual components by connecting some point along the string to the guard and making a three-terminal measurement. Sometimes three-terminal measurements are simply called guarded measurements. They are also called direct impedance measurements. Figure 5 illustrates one representation of a passive 3-terminal network.



$$Z_m = \frac{V}{I} \cong Z_x \left( 1 + \frac{z_1 + z_3}{Z_x} + \frac{z_1}{Z_a} + \frac{z_3}{Z_b} - \frac{z_5 Z_x}{Z_a Z_b} \right)$$

**Figure 5: 3-Terminal Measurement**

The impedance  $Z_x$  is that impedance directly between points A and B. As shown by the formula for Figure 5, errors caused by  $Z_a$  and  $Z_b$  have been changed. If it were not for the series impedances, the effect of  $Z_a$  and  $Z_b$  would have been removed completely. The combination of series impedance and shunt impedance has given us two new types of errors. We'll call the first ( $z_1/Z_a$  and  $z_3/Z_b$ ) the "series/shunt" error. It's caused by a voltage, or current, divider effect. The voltage between point A and guard is reduced because the attenuating or dividing effect of the impedances  $z_1$  and  $Z_a$ . Likewise,  $Z_b$  and  $z_3$  divide the current  $I_x$  so that it doesn't all flow in the ammeter. Note that this error is a constant percent error, independent of the value of  $Z_x$ . It usually is very small at low frequencies unless the series and shunt impedances are actual circuit components as they might be in in-circuit measurements.

## Guard Lead Error

The second new type of error (the last term) is the "guard-lead error". It can become important when  $Z_x$  is very large, but it is (generally) much smaller than the shunt error of Figures 2 and 3 because the ratio  $z_5/Z_a$  is usually very small. This error can most easily be explained by making a Y to delta transformation of the Y network formed by  $Z_a$ ,  $Z_b$ , and  $z_5$ . The result gives an impedance value:

$$Z_a + Z_b + \frac{Z_a Z_b}{z_5} \quad (\text{directly across } Z_x).$$

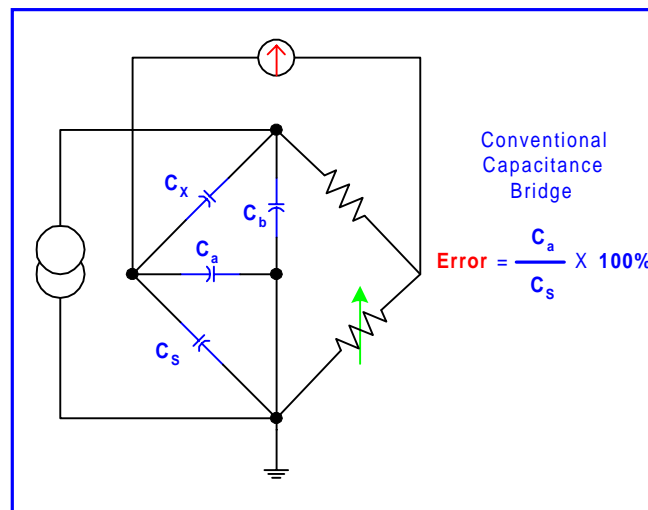
Since the last term is by far the largest ignore the first two and thus the shunt error term is:

$$\frac{-z_5 Z_x}{Z_a Z_b}$$

One situation in which the guard lead error is noticeable is the precision measurement of low-loss 3-terminal capacitors using long, shielded leads. Here the impedances  $Z_a$  and  $Z_b$  are capacitances ( $C_a$  and  $C_b$ ) that can be quite large. Any resistance in the guard connection ( $r_5$ ) causes a negative resistance across the capacitor being measured:

$$\frac{Z_a Z_b}{z_5} = \frac{-1}{\omega^2 C_a C_b r_5}$$

This can give a negative conductance or dissipation factor bridge reading that has bewildered many a metrologist. For in-circuit testing, this error can be very important; for example, if  $Z_a$  and  $Z_b$  are  $100\Omega$  resistors, the effective impedance shunting  $Z_x$  is  $10k\Omega$  if the lead impedance is only  $10m\Omega$ . Many traditional four-arm capacitance bridges (such as that of Figure 6a) make fair three-terminal measurements because they will tolerate some loading across the standard capacitor without excessive error and because loading across the source (or detector) causes no errors directly (it only reduces sensitivity).

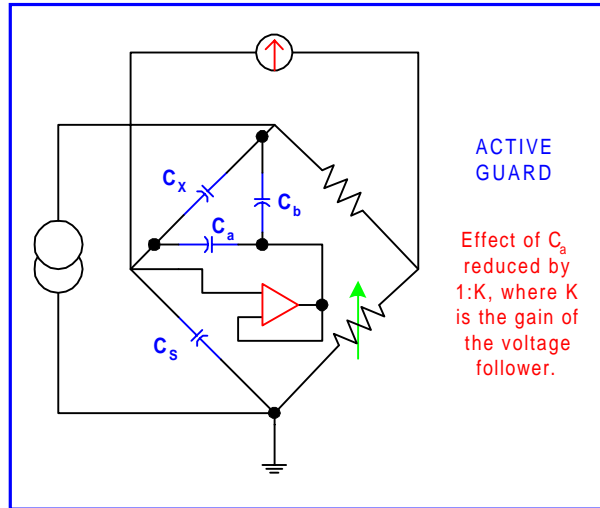


**Figure 6a: Conventional Capacitance Bridge**

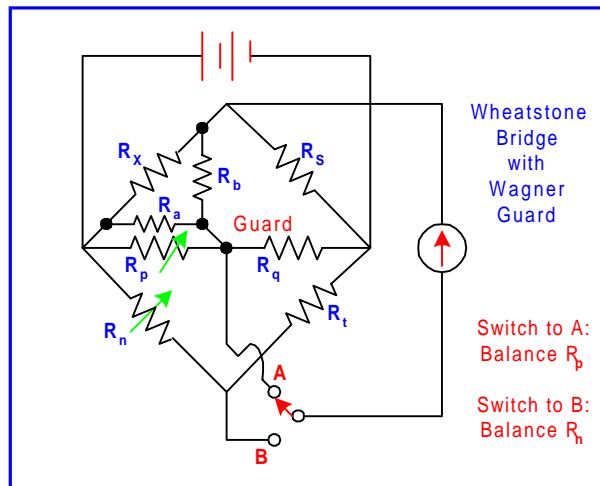


## Active & Wagner Guard Measurements

In QuadTech's general-purpose bridges, the standard is 0.1 $\mu$ F so that 100pF loading makes only a 0.1% error. A unity-gain amplifier could be added to make an active guard (Figure 6b). Here, there is no voltage across  $C_a$  if the gain is exactly unity so that no current flows through it and causes no error. Another way to remove the effect of shunt loading is to use a "Wagner guard" such as that shown in the resistance bridge of Figure 6c.



**Figure 6b: Active Guard**

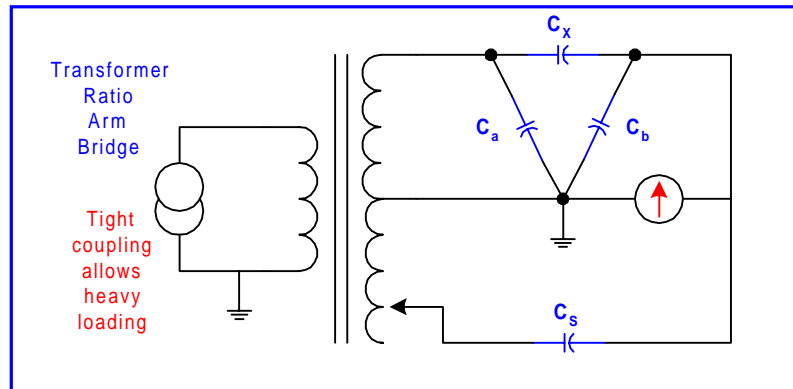


**Figure 6c: Wheatstone Bridge with Wagner Guard**

If the auxiliary balance ( $R_p$ ) is made,  $R_a$  and  $R_b$  have no effect. Similar guard circuits have been used in commercial bridges.

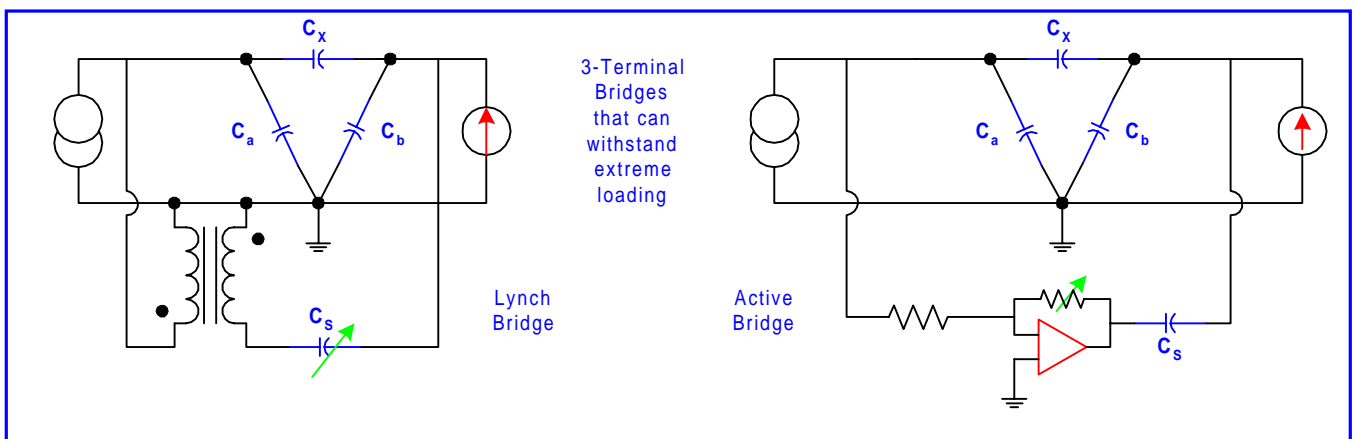
## Three-Terminal Bridge Measurements

However, most precision AC bridges now use tightly coupled transformers (Figure 6d) to give both precise ratios and good immunity from shunt loading. If the voltage of one output winding of the transformer is reduced loading, the voltage on the other is reduced also because of the coupling between them.



**Figure 6d: Transformer Ratio Arm Bridge**

The effective output impedance of each winding is only its resistance and the leakage inductance, and these can be very small if the transformer is properly designed. A modified transformer connection (Figure 6e) has no loading errors because  $C_a$  shunts the source and  $C_b$  the detector so that they cause only a loss of sensitivity. An analogous active bridge is shown in Figure 6f. The accuracy of both these bridges depends on the accuracy of the inverting device, transformer or op amp inverter.

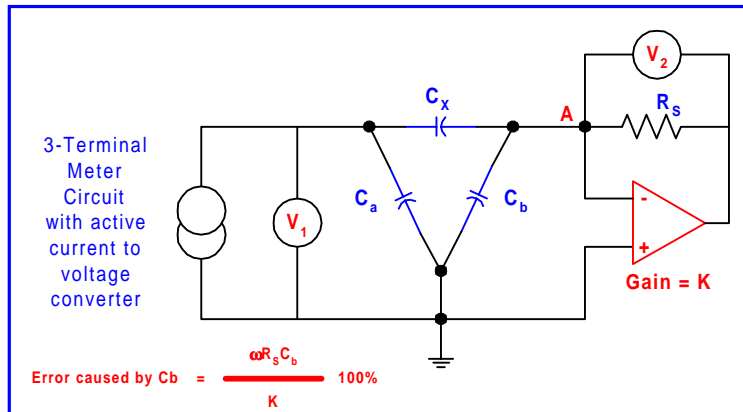


**Figure 6e: Lynch Bridge**

**Figure 6f: Active Bridge**

## Three-Terminal Measurements

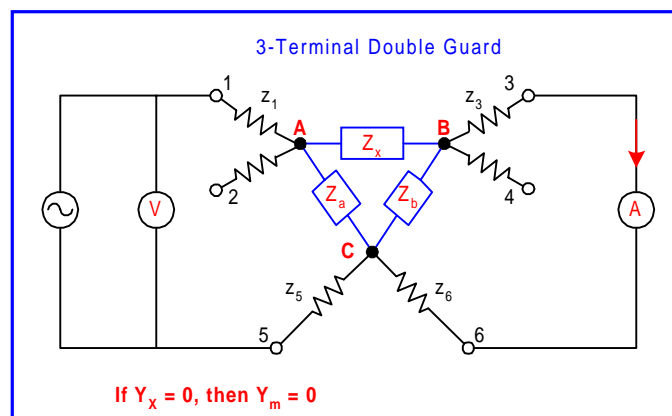
QuadTech's newer digital meters use an inverting amplifier and resistor as a current-to-voltage converter (Figure 6g). This amplifier puts its input (A) to a "virtual ground", that is, it brings this point very nearly to ground potential. The effective impedance to ground is the feedback resistance ( $R_s$ ) divided by the amplifier open-loop gain. The error from shunting capacitance ( $C_b$ ) can be quite small.



**Figure 6g: 3-Terminal Meter Circuit**

### Three-Terminal, Double-Guard-Lead Connection

The guard lead error of Figure 5 can be removed by using two guard connections as illustrated in Figure 7. (This results in a total of four connections, but let's not call it a four-terminal measurement. We'll reserve that for the "Kelvin" type measurement.)

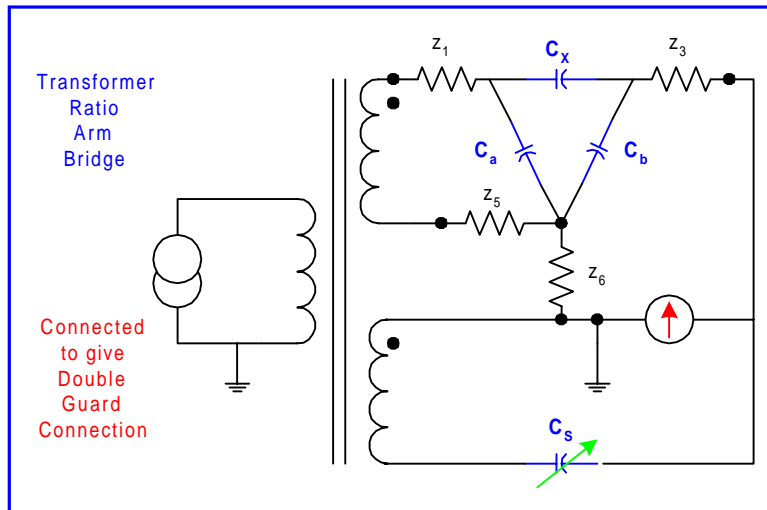


$$Z_m = \frac{V}{I} \cong Z_x \left( 1 + \frac{Z_1 + Z_3 + Z_5 + Z_6}{Z_x} + \frac{Z_1 + Z_5}{Z_a} = \frac{Z_3 + Z_6}{Z_b} \right)$$

**Figure 7: 3-Terminal, Double Guard**

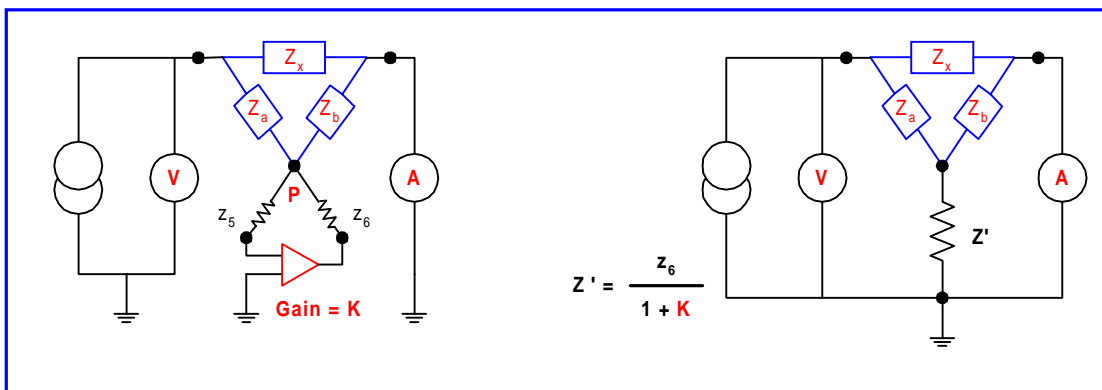
## More Double Guard Measurements

The 'double guard' connection simply puts the guard lead impedances in series with the other lead impedances. There is no shunt error at all and therefore this method is excellent for very high impedance measurements. A possible capacitance bridge making use of the double guard lead is shown in Figure 8a. This circuit takes advantage of the isolating ability of a transformer as well as its other features. It puts one guard impedance,  $z_5$ , in series with  $z_1$  and the other,  $z_6$ , in series with  $z_3$ .



**Figure 8a: Transformer Ratio Arm Bridge in Double Guard Connection**

The active circuit of Figure 8b is used in a DC bridge used in large systems that have relatively high lead impedances because the connections are switched and the wires are long. The inverting amplifier puts point P at a virtual ground. This effectively reduces the guard lead impedance by the gain of the amplifier.



**Figure 8b: Active Guard Circuit**

## More Double Guard Measurements

The meter circuit of Figure 6g can form a double guard lead circuit if the grounding is modified to allow two guard connections. This may be accomplished by adding an isolating transformer (Figure 8c) or simply by lifting the amplifier ground, thus putting  $z_6$  in series with this high impedance input (Figure 8d).

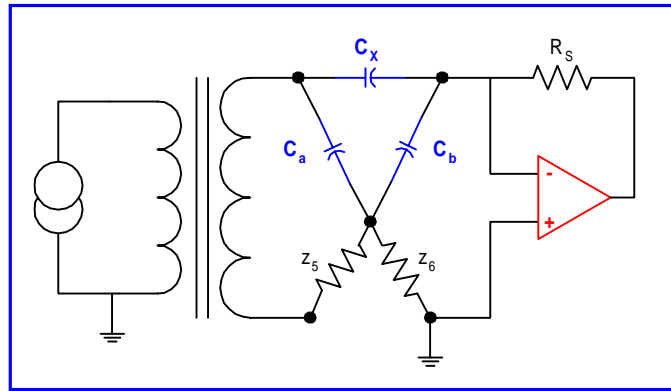


Figure 8c: Isolating Transformer

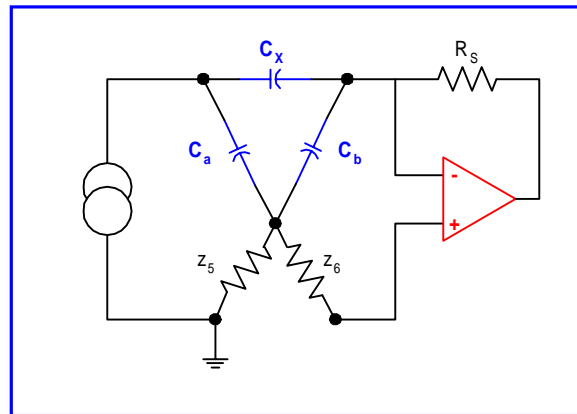
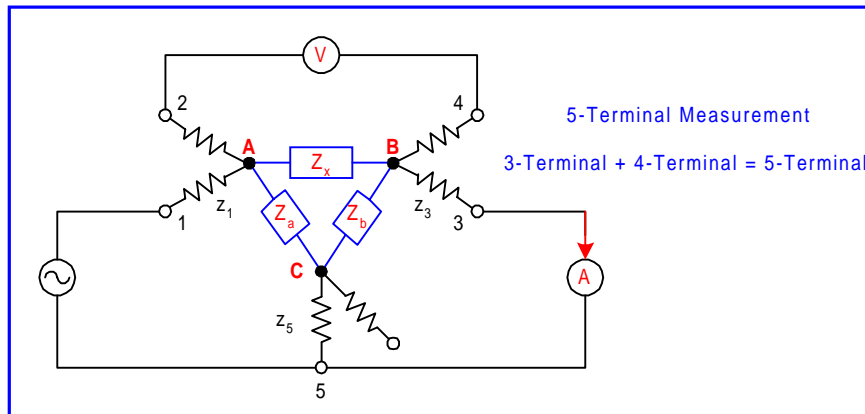


Figure 8d: Modified Inverter

## Five-Terminal Measurements

When both the three-terminal and four-terminal techniques are combined, as in Figure 9, the result is a total of five terminals, **not** seven. The term "five terminal" is not widely used, but it should be more common in the future because most new measuring instruments have this capability. The main reason they have it is so that they can make measurements over a very wide range of impedances. They need the four-terminal capability for low impedances and three-terminal capability for high impedances, particularly small capacitances. It is relatively rare to require both capabilities for a single measurement except for extremely precise measurements, remote measurements, or in-circuit measurements. This technique has neither of the original two-terminal errors and is left with only much smaller errors caused by the interaction of series and shunt impedances.

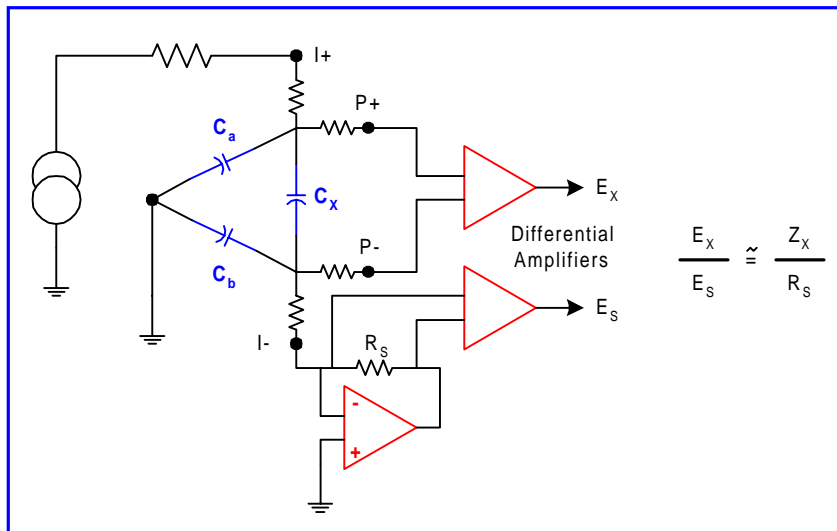
## Five Terminal Measurements



$$Z_m = \frac{V}{I} \cong Z_x \left( 1 + \frac{z_3}{Z_b} - \frac{z_5 Z_x}{Z_a Z_b} \right)$$

**Figure 9: Five-Terminal Measurement**

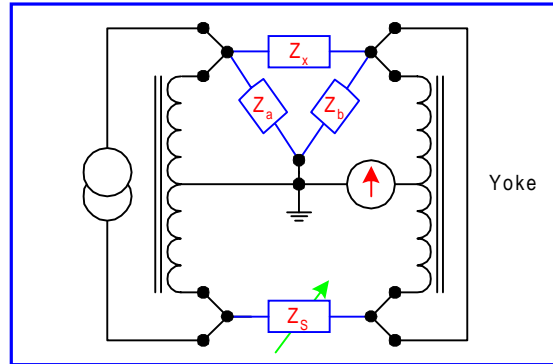
The QuadTech digital impedance meters make a five-terminal measurement by means of the circuit of Figure 10a. This uses the differential amplifiers and the inverter in previous circuits.



**Figure 10a: Active 5-Terminal Meter Circuit**

## Five Terminal Measurements

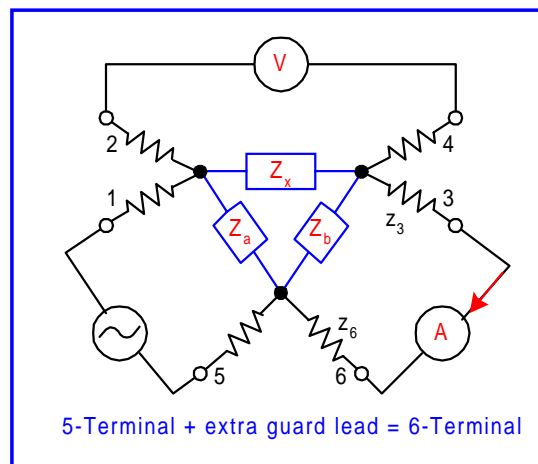
A passive five-terminal bridge is shown in Figure 10b. This is an AC version of the 4-terminal Kelvin bridge that uses transformers whose effective low impedance provides good three-terminal capability.



**Figure 10b: Hill's Transformer Bridge (AC Version of Kelvin Bridge)**

## Six-Terminal Measurements

The next step is obvious. If we use the double-guard technique with the five-terminal method, we have a "six-terminal" measurement that avoids guard lead errors (Figure 11).

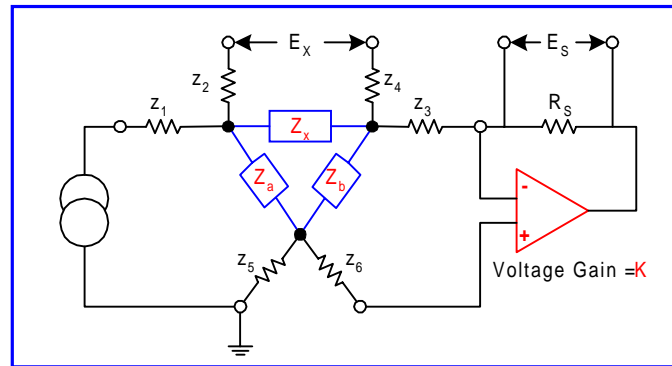


$$Z_m = \frac{V}{I} = Z_x \left( 1 + \frac{Z_3 + Z_6}{Z_b} \right)$$

**Figure 11: 6-Terminal Measurement**

## Six Terminal Measurements

The only error left is a series/shunt error which was the least important type of error of all those considered. Six-terminal measurement circuits can be formed by adding the active circuit of Figure 8b to any five-terminal circuit or by modifying the active five-terminal circuit of Figure 10a by adding an isolation transformer as in Figure 8c or using the modified inverter connection of Figure 8d. The latter is shown in Figure 12 along with its error formula.



$$\frac{E_x}{E_s} = \frac{Z_x}{Z_s} \left( 1 + \frac{z_3}{Z_b} + \frac{R_s}{(1+K)Z_b} - \frac{z_5 Z_x}{(1+K)Z_a Z_b} \right)$$

**Figure 12: 6-Terminal Method**

Figure 12 combines 3-terminal double guard and 4-terminal methods. Only a very few instruments make use of a six-terminal connection. An application where the six-terminal connection is particularly useful is for testing components assembled on a printed circuit board.

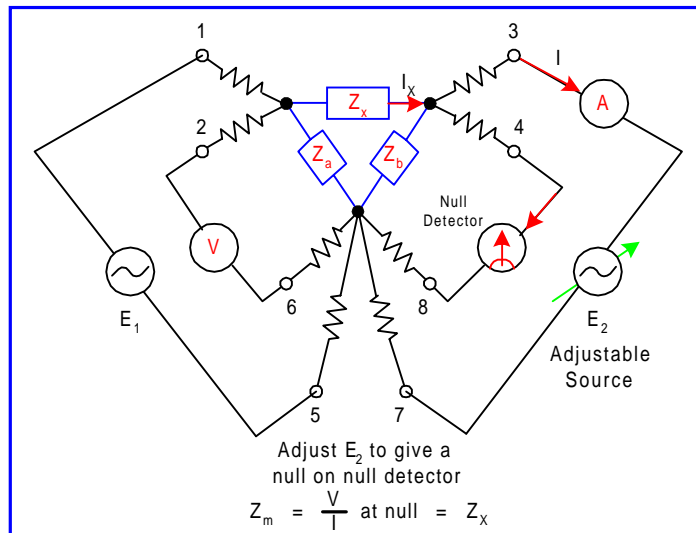
### How About Eight Terminals?

While the five-and six-terminal methods should be adequate for almost everyone, there is always somebody who is not satisfied. At the National Institute of Standards and Technology they talk about accuracies better than a part in  $10^8$ , and to get down there requires even more terminals. They have developed a four-terminal-pair method (outlined in Figure 13), and four pairs equals eight terminals. Our previous measurements use imaginary meters. In this one we have to make imaginary adjustment as well. If one were to adjust a second voltage,  $E_2$ , (synchronous with  $E_1$ ) until the null detector indicated zero voltage, then no voltage appears across  $Z_b$  so that the ideal voltmeter reads the voltage across  $Z_x$  exactly, and no current flows in  $Z_b$  or in the null detector so that all the current  $I_x$  flows in the ammeter.



## Eight Terminal Measurements

The ratio of the reading is  $Z_x$  exactly with no error. The NIST system uses shielded coaxial connections to avoid errors caused by mutual inductance and electrostatic and electromagnetic pickup. These shielded leads have effective shunt impedance at both ends while in our test circuit  $Z_a$  and  $Z_b$  represent only shunt impedance at the ends of the cables tied to  $Z_x$ . If impedance at the other ends of the cables is added, this method has slight series/shunt type errors as indicated by the formula given in Figure 13. (Here  $Z_{26}$  is between terminals #2 and #6, etc.). These errors are relatively small and, moreover, can be easily determined and accounted for.

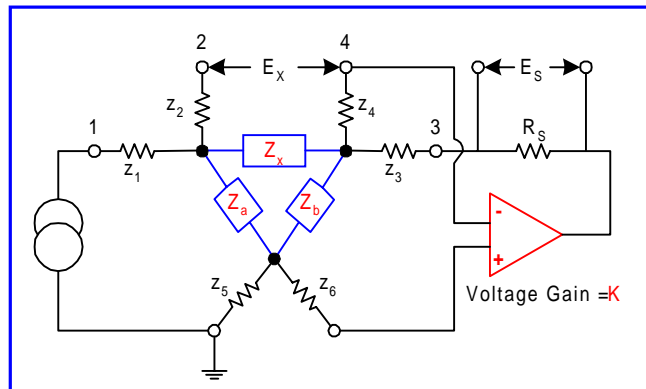


$$Z_m = Z_x \left( 1 - \frac{Z_2 + Z_6}{Z_{26}} - \frac{Z_3 + Z_7}{Z_{37}} \right)$$

**Figure 13: 8- Terminal Measurement (4-Terminal-Pairs)**

While a realization of this coaxial, four-pair method in actual hardware is beyond the intended scope of this note, a modified, six-terminal version of the method is practical. To obtain this circuit (Figure 14) one should note the following changes from Figure 13: 1) At null, there is no current in connections #6 and #8 so that they can be combined without error. 2) Voltage drops in leads #5 and #7 have no effect so that these leads may be combined. 3) The voltmeter could be just as well be placed between terminals #2 and #4, (terminals 6, 8, and 4 are all at the same potential at null). And 4) an op amp can replace the null detector-adjustable source combination because, with negative feedback, the output of an op amp brings its input to (almost) zero volts (i.e., a "null"). The result, Figure 14, is very close to the six-terminal circuit of Figure 12, the differences being the inclusion of  $z_3$  in the feedback path and the dual use of terminal #4. Note that for the new circuit, as drawn, there is no error if the op amp has infinite gain.

## Alternate 6-Terminal Measurement



$$\frac{E_x}{E_s} = \frac{Z_x}{Z_s} \left( 1 + \frac{z_3}{Z_b} + \frac{R_s}{(1+K)Z_b} - \frac{z_5 Z_x}{(1+K)Z_a Z_b} \right)$$

**Figure 14: Alternate 6-Terminal Circuit**

### Conclusion

While some of the ideal methods shown were completely immune to series or shunt impedances, actual measuring instruments can never be. Nothing real is perfect. The degree of immunity of actual circuits varies widely as does the degree of immunity required for different applications. A precision resistance bridge which has negligible error from  $1\Omega$  in each lead is a good four-terminal bridge for measuring resistors because test leads and connections usually have much less resistance than  $1\Omega$ . Likewise a capacitance bridge immune to  $1000\text{pF}$  of shunt loading is perfectly adequate for three-terminal measurements on capacitors. If the unknown is remotely located from the measuring instrument so that long leads are required, both series resistance and shunt capacitance will increase, and better immunity is required. The most difficult multi-terminal measurements are in-circuit measurements where the series and shunt impedances are actual circuit components that may be of any value. It is easy to imagine component values in the test circuit of Figure 1 that would make it impossible to measure  $Z_x$  accurately with any available instrument. However, instruments are available that can measure all but a few percent of the components in a typical printed circuit board. Because different applications require different degrees of immunity, simply saying that an instrument has three-terminal or four-terminal capability is not sufficient. Some measure of these capabilities should be incorporated in its specifications.

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